J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

SYJC PRELIUM – 01 **SET - A**

DURATION – 3 HR MARKS – 80

SOLUTION SET

SECTION - I

Q1. (A) Attempt any six of the following (12)

Q1

01. $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ Find x and y

SOLUTION $\begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$$\begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

By Equality of Two Matrices

$2 + y = 5$	$2x + 2 = 8$
$y = 3$	$2x = 6$
	$x = 3$

$x = 3$ & $y = 3$

02. Find $\dfrac{d^2y}{dx^2}$ if $y = \log x$

SOLUTION $y = \log x$

Diff. wrt x : $\dfrac{dy}{dx} = \dfrac{1}{x}$

Diff. once again wrt x

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}$$

03. Discuss the continuity of function f at $x = 0$

$$f(x) = \frac{\sqrt{4+x} - 2}{3x} \quad ; \quad x \neq 0$$

$$= 1/12 \quad ; \quad x = 0$$

SOLUTION

Step 1

$$\lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{\sqrt{4+x} - 2}{3x}$$

$$= \lim_{x \to 0} \frac{\sqrt{4+x} - 2}{3x} \cdot \frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2}$$

$$= \lim_{x \to 0} \frac{4 + x - 4}{3x} \cdot \frac{1}{\sqrt{4+x} + 2}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{3\cancel{x}} \cdot \frac{1}{\sqrt{4+x} + 2} \qquad x \neq 0$$

$$= \lim_{x \to 0} \frac{1}{3} \cdot \frac{1}{\sqrt{4+x} + 2}$$

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{4+0} + 2}$$

$$= \frac{1}{3} \cdot \frac{1}{2+2}$$

$$= \frac{1}{12}$$

Step 2 :

$f(0) = 1/12$ ……… given

Step 3 :

$f(0) = \lim_{x \to 0} f(x)$; f is continuous at $x = 0$

04. the total revenue $R = 720x - 3x^2$ where x is number of items sold . Find x for which total revenue R is increasing

SOLUTION $R = 720x - 3x^2$

For Revenue increasing

$$\frac{dR}{dx} > 0$$

$720 - 6x > 0$

$720 > 6x$

$120 > x$

$x < 120$

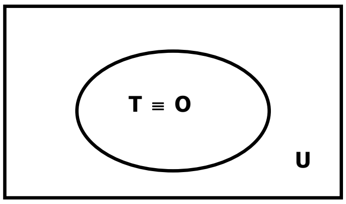
05. Express the truth of each of the following statements using Venn Diagram

1. All teachers are scholars and scholars are teachers

T ≡ set of all teachers

O ≡ set of all scholars

U ≡ set of all human beings

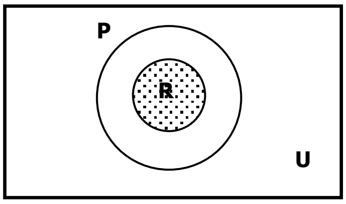


2. if a quadrilateral is a rhombus then it is a parallelogram

R ≡ set of all rhombus

P ≡ set of all parallelograms

U ≡ set of all quadrilaterals



06. Write negations of the following statements

a) the number 6 is an even number or the number 25 is a perfect square

Using $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

Negation : 6 is not an even number and 25 is not a perfect square

b) if $x \in A \cap B$, then $x \in A$ and $x \in B$

Using $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

Negation : $x \in A \cap B$ and $x \notin A$ OR $x \notin B$

07. Evaluate : $\displaystyle\int \frac{1}{x(3 + \log x)}\,dx$

SOLUTION

$3 + \log x = t$

$\frac{1}{x}dx = dt$

$\displaystyle\int \frac{1}{t}\,dt$

$\log | t | + c$

Resubstitute

$\text{Log} \mid 3 + \log x \mid + c$

08. $\left\{ 3 \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 5 & -2 \\ -3 & -4 & 4 \end{pmatrix} \right\} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. Simplify

SOLUTION

$$\left\{ \begin{pmatrix} 3 & 6 & 0 \\ 0 & -3 & 9 \end{pmatrix} - \begin{pmatrix} 1 & 5 & -2 \\ -3 & -4 & 4 \end{pmatrix} \right\} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{pmatrix} 3-1 & 6-5 & 0+2 \\ 0+3 & -3+4 & 9-4 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 5 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{pmatrix} 2 + 2 + 2 \\ 3 + 2 + 5 \end{pmatrix}$$

Q2. (A) Attempt any TWO of the following **(06)**

01. Solve the following equations by the inversion method

$$2x + 3y = -5 \quad \text{and} \quad 3x + y = 3$$

STEP 1 :

$$\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

$AX = B$

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

STEP 2 :

$AA^{-1} = I$

$$\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$R_1 - R_2$

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & -2 \\ 0 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$R_2/7$

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} A^{-1} = \frac{1}{7}\begin{bmatrix} 7 & -7 \\ -2 & 3 \end{bmatrix}$$

$R_1 + 2R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$$

Q2A

$$I.A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$$

STEP 3 :

$$X = A^{-1}B$$

$$= \frac{1}{7}\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 9 + 5 \\ -6 - 15 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 14 \\ -21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

BY EQUALITY OF MATRICES

$x = 2$ & $y = -3$

02. Using the truth table , examine whether the statement pattern

$$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$$

is a tautology , a contradiction or a contingency

SOLUTION

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$	$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Since all the truth values in the last column are 'T' , the statement is a TAUTOLOGY

03. Demand function x , for a certain commodity is given as x = 200 – 4p where p is the unit price . Find elasticity of demand when p = 10 , interpret your result

SOLUTION

STEP 1 : $x = 200 - 4p$

$$\frac{dx}{dp} = -4$$

STEP 2 :

$$\eta = \frac{-P}{D} \cdot \frac{dD}{dp}$$

$$= \frac{-p}{x} \cdot \frac{dx}{dp}$$

$$= \frac{-p}{200 - 4p} \cdot -4$$

$$= \frac{p}{50 - p}$$

STEP 3 :

$$\eta \Big|_{p = 10} = \frac{10}{50 - 10}$$

$$= \frac{10}{40}$$

$$= 0.25 < 1$$

Demand is relatively inelastic

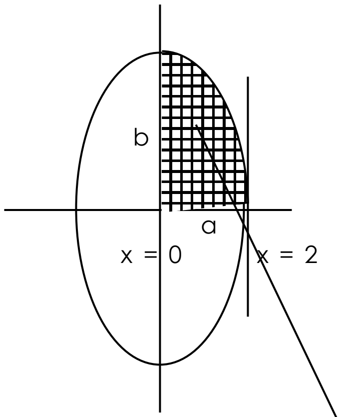
(B) Attempt any TWO of the following **(08)**

Q2B

01. Find the area of the ellipse : $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$

$a^2 = 4 \qquad b^2 = 25$

$a = 2 \qquad b = 5$



$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

$$\frac{y^2}{25} = 1 - \frac{x^2}{4}$$

$$\frac{y^2}{25} = \frac{4 - x^2}{4}$$

$$y^2 = \frac{25}{4}(4 - x^2)$$

$$y = \frac{5\sqrt{4 - x^2}}{2}$$

Area of Ellipse

$$= 4\int_0^2 y \; dx \qquad \text{....... BY SYMMETRY}$$

$$= 4\int_0^2 \frac{5\sqrt{4 - x^2}}{2}\, dx$$

$$= 10\int_0^2 \sqrt{2^2 - x^2}\; dx$$

$$= 10\left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 10\left[\frac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 10\left\{\left[\frac{2}{2}\sqrt{2^2 - 2^2} + 2\sin^{-1}\left(\frac{2}{2}\right)\right] - \left[\frac{0}{2}\sqrt{2^2 - 0^2} + 2\sin^{-1}\left(\frac{0}{2}\right)\right]\right\}$$

$$= 10\left[0 + 2\sin^{-1}(1)\right] - \left[0 + 2\sin^{-1}(0)\right]$$

$$= 10\left(2 \times \frac{\pi}{2}\right)$$

$$= 10\pi \text{ sq. units}$$

02. Evaluate : $\int x.\tan^{-1}x\, dx$

SOLUTION

$$= \int \tan^{-1}x.\ x\, dx$$

$$= \tan^{-1}x \int x\, dx - \int \left[\frac{d}{dx} \tan^{-1}x \int x\, dx \right] dx$$

$$= \tan^{-1}x\ .\frac{x^2}{2} - \int \frac{1}{1+x^2} . \frac{x^2}{2}\, dx$$

$$= \frac{x^2}{2}. \tan^{-1}x - \frac{1}{2}\int \frac{x^2}{1+x^2.}\, dx$$

$$= \frac{x^2}{2}. \tan^{-1}x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2.} . dx$$

$$= \frac{x^2}{2}. \tan^{-1}x - \frac{1}{2}\int 1 - \frac{1}{1+x^2.}\, dx$$

$$= \frac{x^2}{2}. \tan^{-1}x - \frac{1}{2}\left(x - \tan^{-1}x\right) + c$$

$$= \frac{x^2}{2}. \tan^{-1}x - \frac{x}{2} + \frac{\tan^{-1}x}{2} + c$$

03. Cost of assembling x wallclocks is $\left(\frac{x^3}{3} - 40x^2\right)$ and labor charges are 500x .

Find the number of wallclocks to be manufactured for which marginal cost is minimum

SOLUTION

STEP 1 : MARGINAL COST C_M

$$C = \frac{x^3}{3} - 40x^2 + 500x$$

$$C_M = \frac{dC}{dx}$$

$$= x^2 - 80x + 500$$

STEP 2 :

$$\frac{dC_M}{dx} = 2x - 80$$

$$\frac{d^2C_M}{dx^2} = 2$$

STEP 3 :

$$\frac{dC_M}{dx} = 0$$

$$2x - 80 = 0$$

$$x = 40$$

STEP 4 :

$$\left.\frac{d^2C_M}{dx^2}\right|_{x = 20} = 2 > 0$$

Marginal cost is minimum at x = 20

Q3. (A) Attempt any TWO of the following (06)

01. Write Converse – Contrapositive & Inverse statements for the given conditional statement

if the triangles are not congruent then their areas are not equal

Q3A

SOLUTION :

LET $P \rightarrow Q$ $\equiv$ if the triangles are not congruent then their areas are not equal

CONVERSE : $Q \rightarrow P$

If the areas of triangles are not equal then they are not congruent

CONTRAPOSITIVE : $\sim Q \rightarrow \sim P$

If the areas of the triangles are equal then they are congruent

INVERSE : $\sim P \rightarrow \sim Q$

If the two triangles are congruent then their areas are equal

02. find a & b if f(x) is continuous at x = 0

$$f(x) = \frac{e^{2x} - 1}{ax} \quad ; \quad x < 0 \,,\, a \neq 0$$

$$= 1 \quad ; \quad x = 0$$

$$= \frac{\log(1 + 7x)}{bx} \quad ; \quad x > 0 \,,\, b \neq 0$$

SOLUTION

Step 1

$$\lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0} \frac{e^{2x} - 1}{ax}$$

$$= \lim_{x \to 0} \frac{1}{a} \frac{e^{2x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{2}{a} \frac{e^{2x} - 1}{2x}$$

$$= \frac{2.\log e}{a}$$

$$= 2/a$$

STEP 2

$$\lim_{x \to 0+} f(x)$$

$$= \lim_{x \to 0} \frac{\log(1 + 7x)}{bx}$$

$$= \lim_{x \to 0} \frac{1}{b} \frac{\log(1 + 7x)}{x}$$

$$= \lim_{x \to 0} \frac{7}{b} \frac{\log(1 + 7x)}{7x}$$

$$= \frac{7}{b}(1)$$

$$= \frac{7}{b}$$

STEP 3

$f(0) = 1$ ……… given

STEP 4

Since f is continuous at $x = 0$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0+} f(x) = f(0)$$

$$\frac{2}{a} = \frac{7}{b} = 1$$

$\therefore$ $a = 2$ & $b = 7$

03. if $x^{2/3} \cdot y^{5/3} = (x + y)^{7/3}$ then show that $\frac{dy}{dx} = \frac{y}{x}$

SOLUTION

$x^{5/3}y^{2/3} = (x + y)^{7/3}$

taking log on both sides

$$\frac{5}{3}\log x + \frac{2}{3}\log y = \frac{7}{3}\log(x + y)$$

$$5\log x + 2\log y = 7\log(x + y)$$

diff. wrt x

$$\frac{5}{x} + \frac{2}{y}\frac{dy}{dx} = \frac{7}{x + y}\frac{d}{dx}(x + y)$$

$$\frac{5}{x} + \frac{2}{y}\frac{dy}{dx} = \frac{7}{x + y}\left(1 + \frac{dy}{dx}\right)$$

$$\frac{5}{x} + \frac{2}{y}\frac{dy}{dx} = \frac{7}{x + y} + \frac{7}{x + y}\frac{dy}{dx}$$

$$\left(\frac{2}{y} - \frac{7}{x + y}\right)\frac{dy}{dx} = \frac{7}{x + y} - \frac{5}{x}$$

$$\frac{2x + 2y - 7y}{y(x + y)}\frac{dy}{dx} = \frac{7x - 5x - 5y}{x(x + y)}$$

$$\frac{2x - 5y}{y(x + y)}\frac{dy}{dx} = \frac{2x - 5y}{x(x + y)}$$

$\frac{dy}{dx} = \frac{y}{x}$ ……. PROVED

(B) Attempt any TWO of the following **(08)**

Q3B

01. $\displaystyle\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt[3]{\cot x}}\,dx$

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt[3]{\cot x}}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx \quad \ldots (1)$$

USING $\displaystyle\int_a^b f(x)\,dx = \int_b^b f(a+b-x)\,dx$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin(\pi/2-x)}}{\sqrt[3]{\sin(\pi/2-x)}+\sqrt[3]{\cos(\pi/2-x)}}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}\,dx \quad \ldots (2)$$

(1) + (2)

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$2I = \left(x \right)_{\pi/6}^{\pi/3}$$

$$2I = \frac{\pi}{3} - \frac{\pi}{6}$$

$$2I = \frac{2\pi-\pi}{6}$$

$$I = \frac{\pi}{12}$$

02. $\int_0^{\pi/2} x^2.\sin x \ dx$

$$= \left\{ x^2 \int \sin x \ dx - \int \left(\frac{d}{dx} x^2 \int \sin x \ dx \right) dx \right\}_0^{\pi/2}$$

$$= \left\{ x^2 . -\cos x - \int 2x . -\cos x \ \ dx \right\}_0^{\pi/2}$$

$$= \left\{ -x^2 . \ \cos x + 2 \int x . \cos x \ dx \right\}_0^{\pi/2}$$

$$= \left\{ -x^2 . \ \cos x + 2 \left(x \int \cos x \ dx - \int \left(\frac{d}{dx} \ x \int \cos x \ dx \right) \ dx \right) \right\}_0^{\pi/2}$$

$$= \left\{ -x^2 . \ \cos x + 2 \left(x . \sin x \ - \int 1. \sin x \ \ dx \right) \right\}_0^{\pi/2}$$

$$= \left\{ -x^2 . \ \cos x + 2 \left(x . \sin x \ - \int \sin x \ dx \right) \right\}_0^{\pi/2}$$

$$= \left\{ -x^2 . \ \cos x + 2 \left(x . \sin x \ + \cos x \right) \right\}_0^{\pi/2}$$

$$= \left\{ -x^2 . \ \cos x \ + \ 2x.\sin x \ + \ 2\cos x \right\}_0^{\pi/2}$$

$$= \left(\frac{-\pi^2}{4} . \cos \frac{\pi}{2} + 2 \frac{\pi}{2} . \sin \frac{\pi}{2} + 2 \cos \frac{\pi}{2} \right) - \left(-0 . \cos 0 + 2(0).\sin 0 + 2 \cos 0 \right)$$

$$= \left(0 + \pi (1) + 0 \right) - \left(0 + 0 + 2(1) \right)$$

$$= \pi - 2$$

03. Express the following equations in matrix form and solve them by method of inversion

$$2x - y + z = 1 \quad ; \quad x + 2y + 3z = 8 \quad ; \quad 3x + y - 4z = 1$$

$$2x - y + z = 1$$

$$x + 2y + 3z = 8$$

$$3x + y - 4z = 1$$

STEP 1 :

$$\begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 1 \end{pmatrix}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

STEP 2 :

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{pmatrix}$$

COFACTOR'S

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = 1(-8 - 3) = -11$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 3 \\ 3 & -4 \end{vmatrix} = -1(-4 - 9) = 13$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1(1 - 6) = -5$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 1 & -4 \end{vmatrix} = -1(4 - 1) = -3$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 1 \\ 3 & -4 \end{vmatrix} = 1(-8 - 3) = -11$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = -1(2 + 3) = -5$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} = 1(-3 - 2) = -5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -1(6 - 1) = -5$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 1(4 + 1) = 5$$

COFACTOR MATRIX OF A

$$\begin{pmatrix} -11 & 13 & -5 \\ -3 & -11 & -5 \\ -5 & -5 & 5 \end{pmatrix}$$

ADJ A **=** TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} -11 & -3 & -5 \\ 13 & -11 & -5 \\ -5 & -5 & 5 \end{pmatrix}$$

|A|

$$= 2(-8 - 3) + 1(-4 - 9) + 1(1 - 6)$$

$$= 2(-11) + 1(-13) + 1(-5)$$

$$= -22 - 13 - 5 = -40$$

$$\mathbf{A^{-1}} = \frac{1}{|A|} . \text{adj } A$$

$$= -\frac{1}{40}\begin{pmatrix} -11 & -3 & -5 \\ 13 & -11 & -5 \\ -5 & -5 & 5 \end{pmatrix}$$

$$= \frac{1}{40}\begin{pmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{pmatrix}$$

STEP 3 :

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{40}\begin{pmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 8 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{40}\begin{pmatrix} 11 + 24 + 5 \\ -13 + 88 + 5 \\ 5 + 40 - 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{40}\begin{pmatrix} 40 \\ 80 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

BY EQUALITY OF TWO MATRICES

$x = 1$, $y = 2$ & $z = 1$ SS : {1, 2 , 1}

SECTION – II

Q4. (A)Attempt any six of the following **(12)**

Q4

01. the pdf of continuous random variable X is given by

$$f(x) = \frac{x}{8} \quad ; \quad 0 < x < 4$$

$$= 0 \quad ; \quad \text{otherwise} \quad . \quad \text{Find } P(2 < X < 3)$$

SOLUTION :

the pdf of continuous random variable X is given by

$$f(x) = \frac{x}{8} \quad ; \quad 0 < x < 4$$

$$= 0 \quad ; \quad \text{otherwise} \quad .$$

$P(2 < X < 3)$

$$= \int_2^3 \frac{x}{8}\, dx$$

$$= \left[\frac{x^2}{16}\right]_2^3$$

$$= \left(\frac{9}{16}\right) - \left(\frac{4}{16}\right)$$

$$= \frac{5}{16}$$

02. What is the sum due of ₹ 5,000 due 4 months hence at 12.5% p.a. simple interest

SOLUTION :

SD = PW + Interest on PW

$$SD = 5000 + 5000 \times \frac{12.5}{100} \times \frac{4}{12}$$

$$= 5000 + 208.33$$

$$= ₹\ 5208.33$$

03. for a bivariate data $b_{yx} = -1.2$ and $b_{xy} = -0.3$.

Find correlation coefficient between x and y

SOLUTION :

$$r^2 = byx \times bxy$$

$$= -\frac{12}{10} \times -\frac{3}{10} \qquad r = -\frac{6}{10} \text{ (byx \& bxy are –ve)}$$

$$= \frac{36}{100}$$

04. Anandi and Rutuja invested ₹ 10,000 each in a business . Anandi withdrew her capital after 7 months . Rutuja continued for the year . After one year the profit earned by them was ₹ 5,700 . Find the profit by each person

SOLUTION :

Profits will be shared in the ratio of 'PERIOD OF INVESTMENT'

PSR : Anandi Rutuja

7 : 12

Anandi's share in profit $= \frac{7}{19} \times 5700 =$ ₹ 2100

Rutuja's share in profit $= \frac{12}{19} \times 5700 =$ ₹ 3600

05. Compute the age specific death rate for the following

SOLUTION :

Age Group	No. of persons In '000	No. of deaths	SDR $= \frac{D}{P}$ (DEATHS PER 000)
Below 5	15	360	$\frac{360}{15} = 24$
5 – 30	20	400	$\frac{400}{20} = 20$
Above 30	10	280	$\frac{280}{10} = 28$

06.

X = x	–1	0	1
P(x)	–0.2	1	0.2

Verify whether the above function can be regarded as p.m.f.

SOLUTION :

P(–1) = –0.2 Since $p(x) \geq 0 \ \forall x$, the function is not a pmf

07. **SOLUTION :**

a) MARGINAL FREQUENCY DISTRIBUTION OF AGE OF HUSBANDS

CI	20 – 30	30 – 40	40 – 50	50 – 60	TOTAL
F	5	20	44	24	93

b) CONDITIONALMARGINAL FREQUENCY DISTRIBUTION OF AGE OF HUSBANDS WHEN AGE WIVES LIES IN 25 – 35

CI	20 – 30	30 – 40	40 – 50	50 – 60	TOTAL
F	0	10	25	2	37

08. from the regression equations : $y = 4x - 5$ and $3x = 2y + 5$.

find $\bar{x}$ and $\bar{y}$ ans : $\bar{x} = 1$ & $\bar{y} = -1$

Q5. (A) Attempt any Two of the following **(06)**

01. From a lot of 25 bulbs of which 5 are defective a sample of 5 bulbs was drawn at random with replacement . Find the probability that the sample will contain

a) exactly 1 defective bulb b) at least 1 defective bulb

Q5A

SOLUTION

a lot of 25 bulbs : 5 defective , 20 non defective

5 bulbs are drawn at random with replacement , n = 5

For a trial Success – a defective bulb

p – probability of success = 5/25 = 1/5

q – probability of failure = 1 – 1/5 = 4/5

r.v. X – no of successes = 0 , 1 , 2 , 3 , 4 , 5

$X \sim B(5 , 1/5)$

a) P(exactly 1 defective bulb)

$= P(X = 1)$

$= {}^5C_1 \cdot p^1 \cdot q^4$

$= {}^5C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^4$

$= \frac{5.4^4}{5^5}$

$= \frac{256}{625}$

b) P(at least 1 defective bulb)

$= P(X \geq 1)$

$= P(1) + P(2) + \ldots\ldots\ldots\ldots + P(5)$

$= 1 - P(0)$

$= 1 - {}^5C_0 \cdot p^0 \cdot q^5$

$= 1 - {}^5C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^5$

$= 1 - \frac{4^5}{5^5}$

$= 1 - \frac{1024}{3125}$

$= \frac{2101}{3125}$

02. The defects on a plywood sheet occur at random with an average of one defect per 50 sq. feet . What is the probability that such sheet will have

a) no defects b) at least one defect

(Given : $e^{-1} = 0.3678$)

SOLUTION

m = average number of defects in a ply wood sheet = 1

r.v X ~ P(1)

P(sheet will have no defects)

= P(0)

$= \frac{e^{-1} 1^0}{0!}$ Using $P(x) = \frac{e^{-m}.m^x}{x!}$

$= e^{-1}. (1)$

= 0.3678

P(sheet will have at least 1 defect)

= $P(x \geq 1)$

= P(1) + P(2) + ...

= 1 – P(0)

= 1 – 0.3678

= 0.6322

03. Find the true discount , banker's discount and banker's gain on a bill of ₹ 36,600 due 4 months hence at 5% p.a.

SOLUTION

STEP 1 :

FV = PW + Int on PW for 4 months @ 5% p.a.

$36600 = PW + PW \times \frac{4}{12} \times \frac{5}{100}$

$36600 = PW + \frac{PW}{60}$

$36600 = \frac{61}{60} PW$

$PW = \frac{36600 \times 60}{61}$

= ₹ 36,000

STEP 2 :

TD $= $ Int on PW for 4 months @ 5% p.a.

$$= 36000 \times \frac{4}{12} \times \frac{5}{100}$$

$= ₹\ 600$

STEP 3 :

BD $=$ Int on FV for 4 months @ 5% p.a.

$$= 36600 \times \frac{4}{12} \times \frac{5}{100}$$

$= ₹\ 610$

STEP 4 :

BG $=$ BD – TD

$= 610 - 600$

$= ₹\ 10$

(B) Attempt any Two of the following **(08)**

01. a car valued at ₹ 4,00,000 is insured for ₹ 2,50,000 . The rate of premium is 5% less 20% . How much loss does the owner bear including premium , if the value of the car is reduced to 60% of its original value

Q5B

SOLUTION

Value of car = ₹ 4,00,000

Insured value = ₹ 2,50,000

Rate of premium = 5 % less 20%.

Premium = $\frac{5}{100}$ x 2,50,000

= ₹ 12,500

less 20% disc − 2,500

Net Premium = ₹ 10,000

Since the value of the car is reduced to 60% of its original value , the loss on the car is 40%

Loss = $\frac{40}{100}$ x 4,00,000

= ₹ 1,60,000

Claim = $\frac{\text{insured val.}}{\text{Property val.}}$ x loss

= $\frac{2,50,000}{4,00,000}$ x 1,60,000

= ₹ 1,00,000

Loss	=	1,60,000
Less claim	−	1,00,000
Net loss	=	60,000
Add premium	+	10,000
Net loss Incl. premium	= ₹	70,000

02.

AGE x	lx	dx = lx − lx+1	$qx = \frac{dx}{lx}$	px = 1 − qx	$Lx = \frac{lx + lx+1}{2}$	Tx	$e_x^0 = \frac{Tx}{lx}$
0	1000	1000 − 850 = 150	$\frac{150}{1000} = 0.15$	1 − 0.15 = 0.85	850 + 75 = 925	2495	$\frac{2495}{1000} = 2.495$
1	850	850 − 760 = 90	$\frac{90}{850} = 0.1059$	1 − 0.1059 = 0.8941	760 + 45 = 805	1570	$\frac{1570}{850} = 1.847$
2	760	760 − 360 = 400	$\frac{400}{760} = 0.5264$	1 − 0.5264 = 0.4736	360 + 200 = 560	765	$\frac{765}{760} = 1.007$
3	360	360 − 25 = 335	$\frac{335}{360} = 0.9305$	1 − 0.9305 = 0.0695	25 + 167.5 = 192.5	205	$\frac{205}{360} = 0.5696$
4	25	25 − 0 = 25	$\frac{25}{25} = 1$	1 − 1 = 0	0 + 12. 5 = 12.5	12.5	$\frac{12.5}{25} = 0.5$
5	0	----	----	----	----	----	----

LOG CALCULATIONS FOR 'qx'

LOG 90 − LOG 850	LOG 400 − LOG 760	LOG 335 − LOG 360
1.9542	2.6021	2.5250
− 2.9294	− 2.8808	− 2.5563
AL $\bar{1}.0248$	AL $\bar{1}.7213$	AL $\bar{1}.9687$
0.1059	0.5264	0.9305

LOG CALCULATIONS FOR 'e_x^0'

LOG 1570 − LOG 850	LOG 765 − LOG 760	LOG 205 − LOG 360
3.1959	2.8837	2.3118
− 2.9294	− 2.8808	− 2.5563
AL 0.2665	AL 0.0029	AL $\bar{1}.7555$
1.847	1.007	0.5696

03. a person makes two types of gift items A and B requiring the services of cutter and a finisher . Gift item A requires 4 hours of cutter's time and 2 hours of finisher's time . Gift item B requires 2 hours of cutter's time and 4 hours of finisher's time . The cutter and finisher have 208 hours and 152 hours available time respectively every month . The profit on one gift item of type A is ₹ 75 and on the gift item B is ₹ 125 . Assuming that a person can sell all the gift items produced , determine how many gift items of each type should he make every month to obtain the best returns

TABULATION :

No of units mfg	ITEM A x	ITEM B y	Maximum Available Time
	Time reqd/unit (in hrs)		(in hrs)
CUTTiING	4	2	208
FINISHING	2	4	152
Profit / unit	75/–	125/–	

CONSTRAINT

1. Since cutter has 208 hours , $4x + 2y \le 208$

2. Since finisher has 152 hours , $2x + 4y \le 152$

3. Since x and y are no of units manufactured , cannot be –ve
$x , y \ge 0$

OBJECTIVE FUNCTION

Total profit = $75x + 125y$ (in rupees)

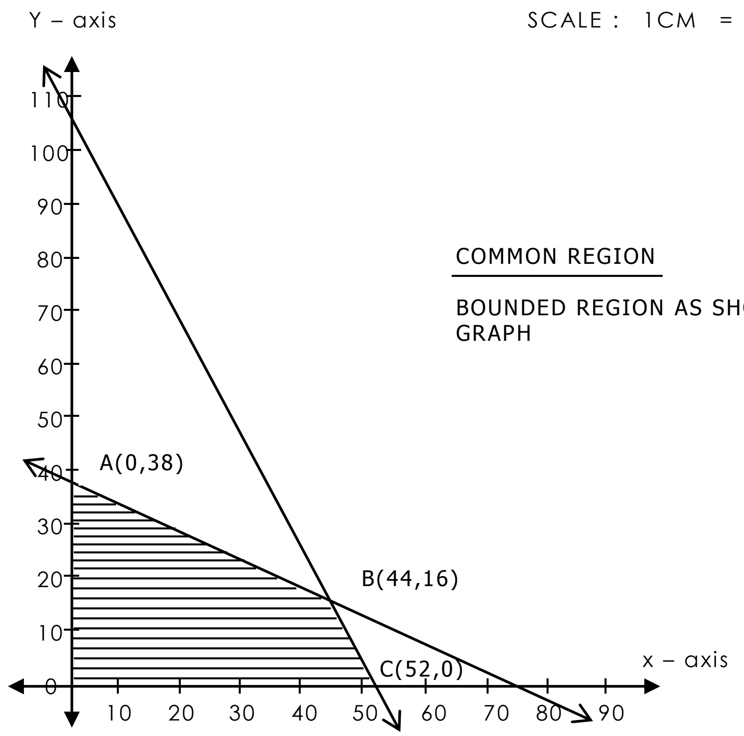
$\therefore$ Maximize $z = 75x + 125y$

LPP MODEL

Maximize $z = 75x + 125y$

Subject to : $4x + 2y \le 208$, $2x + 4y \le 152$, $x , y \ge 0$

SCALE : 1CM = 10 UNITS



COMMON REGION

BOUNDED REGION AS SHOWN IN THE GRAPH

CORNERS	Z = 75x + 125y
O(0,0)	Z = 75(0) + 125(0) = 0
A(0,38)	Z = 75(0) + 125(38) = 4750
B(44,16)	Z = 75(44) + 125(16) = 3300 + 2000 = 5300
C(52,0)	Z = 75(52) + 125(0) = 3900

OPTIMAL SOLUTION

person needs to make 44 units of item A & 16 units of item B to make a maximum Profit of ₹ 5300

Q6. (A) Attempt any Two of the following **(06)**

01. John and Mathew started a business with their capitals in the ratio 8 : 5 . After 8 months , John added 25% of his earlier capital as further investment . At the same time , Mathew withdrew 20% of his earlier capital . At the end of the year , they earned Rs 52,000 as profit . How should they divide the profit between them

Q6A

SOLUTION

PARTNER's NAME	CAPITAL INVESTED	PERIOD OF INVESTMENT
P	₹ 8k	8 MONTHS
+ 25%	₹ 2k	
	₹ 10k	4 MONTHS
Q	₹ 5k	8 MONTHS
−20%	₹ k	
	₹ 4k	4 MONTHS

STEP 1 :

Profits will be shared in the

'RATIO OF PRODUCT OF CAPITAL INVESTED & PERIOD OF INVESTMENT'

	P		Q	
=	8k x 8 + 10k x 4	:	5k x 8 + 4k x 4	
=	64k + 40k	:	40k + 16k	
=	104k	:	56k	
=	13	:	7	TOTAL = 20

STEP 2 :

Total profit = ₹ 52,000

P's share of profit = $\frac{13}{20}$ x 52000

= ₹ 33,800

Q's share of profit = $\frac{7}{20}$ x 52,000

= ₹ 19,200

02. Find mean and variance of the continuous random variable X whose p.d.f is given as

$$f(x) = 6x(1-x) \qquad 0 < x < 1$$
$$= 0 \qquad \text{otherwise}$$

SOLUTION

i) $E(X) = \int_0^1 x.f(x)\,dx$

$= \int_0^1 x.6x(1-x)\,dx$

$= 6\int_0^1 x^2(1-x)\,dx$

$= 6\int_0^1 (x^2 - x^3)\,dx$

$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$

$= 6\left(\frac{1}{3} - \frac{1}{4}\right)$

$= 6\left(\frac{1}{12}\right)$

$= \frac{1}{2}$

$= 0.5$

i) $Var(X) = \int_0^1 x^2.f(x)\,dx - \left(E(X)\right)^2$

$= \int_0^1 x^2.6x(1-x)\,dx - \frac{1}{4}$

$= 6\int_0^1 x^3(1-x)\,dx - \frac{1}{4}$

$= 6\int_0^1 (x^3 - x^4)\,dx - \frac{1}{4}$

$= 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 - \frac{1}{4}$

$= 6\left(\frac{1}{4} - \frac{1}{5}\right) - \frac{1}{4}$

$= 6\left(\frac{1}{20}\right) - \frac{1}{4}$

$= \frac{3}{10} - \frac{1}{4}$

$= \frac{12 - 10}{40}$

$= \frac{1}{20}$

$= 0.05$

03. For bi – variate data $\overline{x} = 53$ and $\overline{y} = 28$, $byx = -1.5$, $bxy = -0.2$

Find a) correlation coefficient between x and y

b) Estimate Y for X = 50 c) Estimate X for Y = 25

SOLUTION

a) Y ON X

$$y - \overline{y} = byx(x - \overline{x})$$

$$y - 28 = -1.5(x - 53)$$

$$y - 28 = -1.5(50 - 53)$$

$$y - 28 = -1.5(-3)$$

$$y - 28 = 4.5$$

$$y = 32.5 \quad \text{for} \quad x = 50$$

a) X ON Y

$$x - \overline{x} = bxy(y - \overline{y})$$

$$x - 53 = -0.2(x - 28)$$

$$x - 53 = -0.2(25 - 28)$$

$$x - 53 = -0.2(-3)$$

$$x - 53 = 0.6$$

$$x = 53.6 \quad \text{for} \quad y = 25$$

(B) Attempt any Two of the following **(08)**

Q6B

01.

		ANTIBIOTICS				
		A	B	C	D	E
	C_1	27	18	---	20	21
CAPSUALTION	C_2	31	24	21	12	17
MACHINES	C_3	20	17	20	---	16
	C_4	21	28	20	16	27

27	18	M	20	21	– add a DUMMY machine C5 to balance the matrix
31	24	21	12	17	
20	17	20	M	16	– M is a very large number such that M – any number = M
21	28	20	16	27	
0	0	0	0	0	

9	0	M	2	3	– Reducing the matrix using 'ROW MINIMUM'
19	12	9	0	5	
4	1	4	M	0	
5	12	4	0	11	
0	0	0	0	0	

9	$\boxed{0}$	M	2	3	– Allocation using 'SINGLE ZERO ROW- COLUMN METHOD'
19	12	9	$\boxed{0}$	5	
4	1	4	M	$\boxed{0}$	
5	12	4	$\cancel{0}$	11	– since allocation is incomplete we need to revise the matrix
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	

9	$\boxed{0}$	M	2	3		– Drawing minimum lines to cover all the existing zeros
19	12	9	$\boxed{0}$	5	√	
4	1	4	M	$\boxed{0}$		
5	12	4	$\cancel{0}$	11	√	
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$		
			√			

9	0	M	6	3	– Revise the matrix
15	8	5	0	1	Reduce all the uncovered elements by its minimum & add the same at the intersection
4	1	4	M	0	
1	8	0	0	7	
0	0	0	4	0	

9	$\boxed{0}$	M	6	3	– Reallocation
15	8	5	$\boxed{0}$	1	– Since each row now contains one
4	1	4	M	$\boxed{0}$	assigned zero , the assignment problem
1	8	$\boxed{0}$	$\cancel{0}$	7	is solved
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	4	$\cancel{0}$	

Optimal Assignment

C_1 – B , C_2 – D , C_3 – E , C_4 – C , C_5 – A (DUMMY) , min cost = 66

02. Regression of two series are

$2x - y - 15 = 0$ & $3x - 4y + 25 = 0$ Find mean of x and y and also the coefficient of correlation

STEP 1

ASSUME

XON Y : $2x - y - 15 = 0$

$2x = y + 15$

$x = \frac{1}{2}y + \frac{15}{2}$

$bxy = \frac{1}{2}$

Y ON X : $3x - 4y + 25 = 0$

$4y = 3x + 25$

$y = \frac{3}{4}x + \frac{25}{4}$

$byx = \frac{3}{4}$

STEP 2

$r^2 = bxy \cdot byx$

$= \frac{1}{2} \times \frac{3}{4}$

$= \frac{3}{8}$

Since $0 \leq r^2 \leq 1$
Our assumptions are correct

$r = \pm\sqrt{\frac{3}{8}}$

$r = +\sqrt{\frac{3}{8}}$ (byx & bxy are + ve)

$\log r = \frac{1}{2}\left(\log 3 - \log 8\right)$

$\log r = \frac{1}{2}\left(0.4771 - 0.9031\right)$

$\log r = \frac{0.4771}{2} - \frac{0.9031}{2}$

$\log r = 0.2386 - 0.4516$

$\log r = \bar{1}.7870$

$r = AL(\bar{1}.7870)$

$r = 0.6124$

STEP 3

MEANS

$2x - y = 15 \times 4$

$3x - 4y = -25$

$8x - 4y = 60$

$3x - 4y = -25$

$5x = 85 \qquad x = 17$

subs in (1) $\qquad y = 19$

03.

x : 9 7 6 8 5

y : 19 17 16 18 15 . Find Karl Pearson's Correlation coeff.

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
9	19	2	2	4	4	4
7	17	0	0	0	0	0
6	16	−1	−1	1	1	1
8	18	1	1	1	1	1
5	15	−2	−2	4	4	4
35 Σx	85 Σy	0 $\Sigma(x-\bar{x})$	0 $\Sigma(y-\bar{y})$	10 $\Sigma(x-\bar{x})^2$	10 $\Sigma(y-\bar{y})^2$	10 $\Sigma(x-\bar{x})(y-\bar{y})$
$\bar{x} = 7$ $\bar{y} = 17$						

$$r = \frac{\Sigma(x-\bar{x}).(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}.$$

$$r = \frac{10}{\sqrt{10} \times \sqrt{10}}$$

$$r = 1$$